Partial test
SCE1106 Control Theory
Tuesday 21. October 2008
kl. 10.15-12.15, Rom F29

The test consists of 4 tasks.
The test counts $15\% = 0.5 * 30$ % of the final grade
in SCE1106 Control with implementation.
The test consists of three pages.
Aid: paper and pen.

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Task 1 (2%): System modeling

Given a system described by the continuous state space model

$$\dot{x} = Ax + Bu \quad (1)$$

$$y = Dx + Eu \quad (2)$$

where

$$A = -0.1, \ \ B = 0.06, \ \ D = 1, \ \ E = -0.1 \quad (3)$$

Show that the sustem can be described by the transfer function model

$$y = h_p(s)u \quad (4)$$

where

$$h_p(s) = k\frac{1 - \tau s}{1 + Ts} \quad (5)$$

Find the gain, k, the time constant, T, and the inverse response time constant, τ, in the transfer function model (5).

Task 2 (12%): PID-control, the Skogestad method

We are going to study a process described by the transfer function model

$$y = h_p(s)u. \tag{6}$$

The process are to be controlled by a controller of the form

$$u = h_c(s)(r - y). \tag{7}$$

The feedback control system is illustrated in Figure (1).

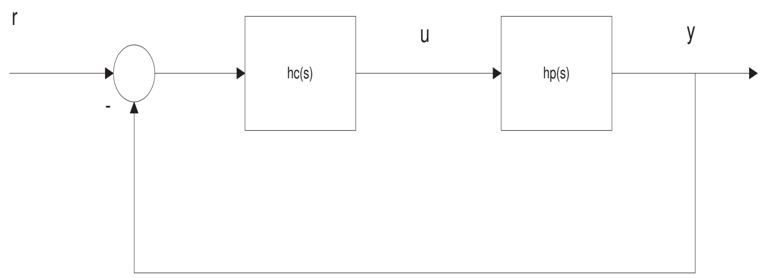

Figure 1: Standard feedback control system.

a) Consider the feedback control system in Figure (1).

- Find the transfer function from the reference, r, to the output measurement, y, i.e., find the transfer function

$$\frac{y}{r} = h_r(s) \tag{8}$$

where $h_r(s)$ is the transfer function from r to y.

- Find an expression for the transfer function, $h_c(s)$, for the controller as a function of the ratio $\frac{y}{r}$ and the transfer function for the process, $h_p(s)$.

We will in the following subtasks specify that the set point response from the reference, r, to the output, y, should be given by

$$\frac{y}{r} = \frac{1 - \tau s}{1 + T_c s} \tag{9}$$

where T_c is a user specified time constant.

b) Suggest a value for the specified time constant T_c for the set point response.

c) Assume that the process, $h_p(s)$, is modelled by a 2nd order transfer function given by

$$h_p(s) = k\frac{1-\tau s}{(1+T_1 s)(1+T_2 s)}, \tag{10}$$

where $T_1 > T_2 > 0$.

Find the controller $h_c(s)$ by the Skogestad method. What type of controller is this?

d) Assume that the process, $h_p(s)$, is modelled by a 1st order model given by

$$h_p(s) = k\frac{1-\tau s}{1+T_1 s}. \tag{11}$$

Find the controller $h_c(s)$ by the Skogestad method. What type of controller is this?

e) Assume that the process, $h_p(s)$, is modelled by a 2nd order oscillating process of the form

$$h_p(s) = k\frac{1-\tau s}{\tau_0^2 s^2 + 2\tau_0 \xi s + 1}. \tag{12}$$

Find the controller $h_c(s)$ by the Skogestad method. What type of controller is this?

f) Assume that the process is modelled by a pure time delay, i.e. with a process model

$$h_p(s) = k e^{-\tau s}. \tag{13}$$

Find the controller $h_c(s)$ by the Skogestad method. What type of controller is this?

g) Assume that the process is modelled by an integrator with time delay, i.e. with a process model

$$h_p(s) = k\frac{e^{-\tau s}}{s}. \tag{14}$$

Find the controller $h_c(s)$ by the Skogestad method. What type of controller is this?

Task 3 (8%): Model reduction and the half rule

a) Given a 5th order process $y = h_p(s)u$ where the process transfer function, $h_p(s)$, is given by

$$h_p(s) = k\frac{1-\tau s}{(1+T_1 s)(1+T_2 s)(1+T_3 s)} \tag{15}$$

where $T_1 \geq T_2 \geq T_3 > 0$.

- Use the half rule for model reduction and find a 1st order model approximation of the form

$$h_p(s) = k\frac{1-\tau s}{1+T_1 s} \tag{16}$$

- Use the half rule for model reduction and find a 2nd order model approximation of the form

$$h_p(s) = k\frac{1-\tau s}{(1+T_1 s)(1+T_2 s)} \tag{17}$$

b) Given the process

$$h_p(s) = k\frac{e^{-\tau s}}{(1+T_0 s)^3} \tag{18}$$

- Use the half rule for model reduction and find a 1st order model approximation of the form

$$h_p(s) = k\frac{1-\tau s}{1+T_1 s} \tag{19}$$

- Use the half rule for model reduction and find a 2nd order model approximation of the form

$$h_p(s) = k\frac{1-\tau s}{(1+T_1 s)(1+T_2 s)} \tag{20}$$

Task 4 (8%): PI control

Consider a PI controller

$$u = h_c(s)e, \tag{21}$$

where e is the controller input, u is the controller output and $h_c(s)$ is the transfer function for the PI controller.

a) Write down the transfer function, $h_c(s)$, of a PI controller.

b) Find a continuous state space equivalent for the PI controller.

c) Use the explicit Euler method and find a discrete time state space equivalent of the PI controller.

d) What is the meaning of ”Anti Wind Up” in connection with the implementation of a PI controller?