Master study
Systems and Control Engineering
Department of Technology
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SCE1106 Control Theory

Exercise 1 b

Exercise 1

This exercise is best worked out at hand and with MATLAB in parallel.
Given a process as described by the following state space model

$$\overbrace{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}}^{\dot{x}} = \overbrace{\begin{bmatrix} 0 & 1 \\ -8 & -6 \end{bmatrix}}^{A} \overbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}^{x} + \overbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}^{B} u \tag{1}$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{D} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x} \tag{2}$$

1. Show that the transfer function of the system is given by

$$h(s) = \frac{1}{s^2 + 6s + 8} \tag{3}$$

   You can check your computation by using the MATLAB Control System Toolbox function **ss2cf.m**.

2. Show that the steady state gain of the process is given 0.125.

3. Show that an eigenvalue matrix of the system is given by

$$\Lambda = \begin{bmatrix} -4 & 0 \\ 0 & -2 \end{bmatrix} \tag{4}$$

4. Show that the time constants of the system is given by $T_1 = 0.5$ and $T_2 = 0.25$.

5. Show that $det(A) =\mid A \mid= 8$.

6. Show that the inverse of the system matrix is given by

$$A^{-1} = \begin{bmatrix} -0.75 & -0.125 \\ 1 & 0 \end{bmatrix} \tag{5}$$

7. Find an eigenvector matrix M corresponding to the eigenvalue matrix Λ. I.e. solve the system of equations $AM = M\Lambda$ or

$$Am_i = \lambda_i m_i \quad i = 1, 2 \tag{6}$$

8. Find the transition matrix of the system $\Phi(t) = e^{At}$.

9. Assume now that the sampling time is $\Delta t = T = 0.1$. Find an exact discrete time model of the form

$$x_{k+1} = A_d x_k + B_d u_k \tag{7}$$

where A_d and B_d is the matrices in the discrete time model equivalent. Use

Make a MATLAB m-file and simulate the response of the system after a step change in the input at time $t = t_0$. Assume that the initial state is zero, i.e.,

$$x(t_0) = x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{8}$$

Use a sampling interval, $\Delta t = 0.1$, and simulate the system over the intervall, $t_0 \leq t \leq t_1$ where $t_0 = 0$ and $t_1 = 5$. Plot the response in MATLAB.

10. Write a m-file script with a **for** loop in order to compute the mean at each time instants $1 \leq i \leq n$ of a variable x_i, i.e., compute the mean

$$\bar{x}_n = \frac{1}{n} \sum_{i=1}^{n} x_i \tag{9}$$